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Topological London theory and bosonization duality in anisotropic topological antiferromagnets

Vira Shyta

Supervisor: Sergei G. Sharapov

Topological states of matter

$$\sigma_{xy} = -\frac{e^2}{2\pi\hbar} C$$

Hall conductance

TKNN invariant integer

(after Thouless, Kohomoto, Nightingale and den Nijs)

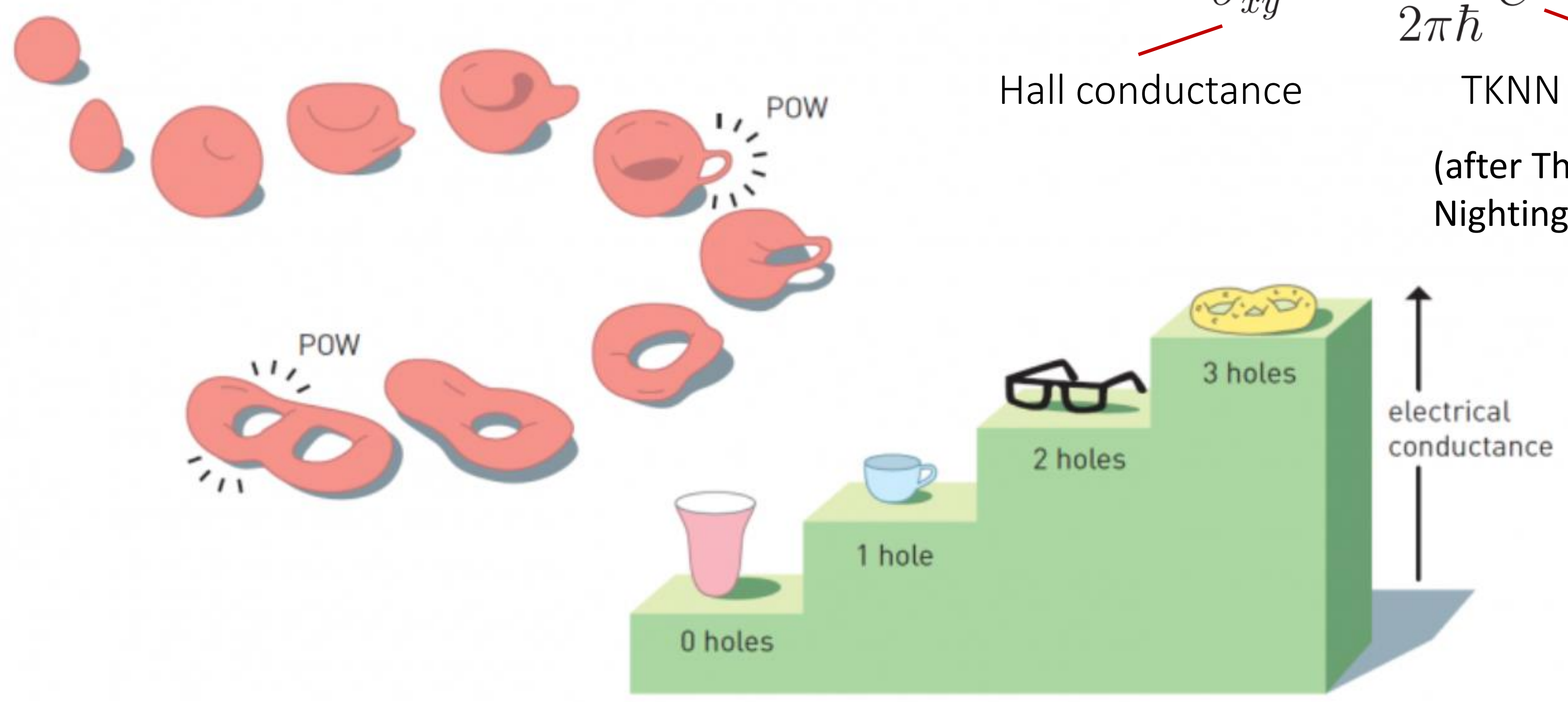


Illustration: by Johan Jarnestad/ The Royal Swedish Academy of Science

Quantum Hall Effect

Hall resistivity: $\rho_{xy} = \left(\frac{h}{e^2}\right)\frac{1}{\nu}$

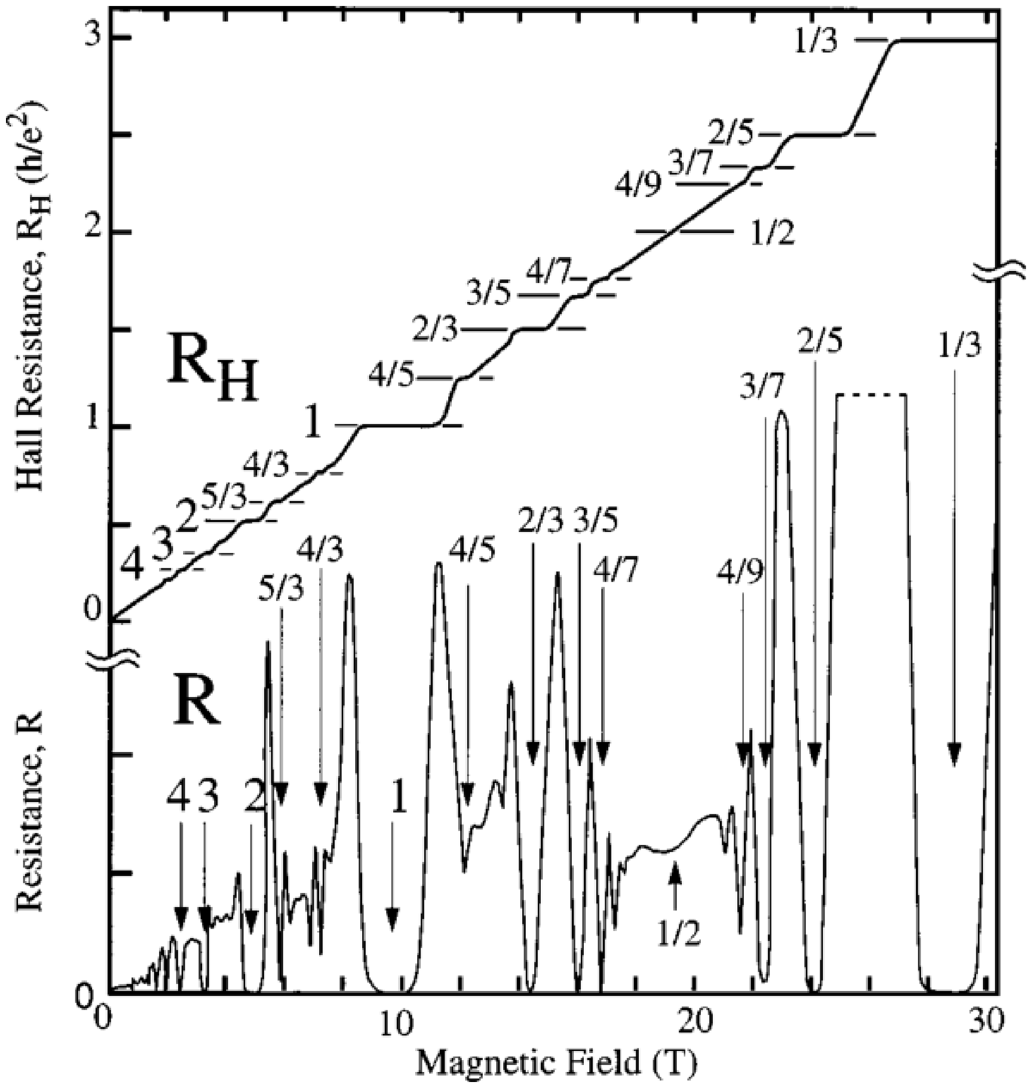
The effective theory of a fractional quantum Hall liquids Lagrangian:

$$\mathcal{L} = -\frac{m}{4\pi} a_\mu \partial_\nu a_\lambda \varepsilon^{\mu\nu\lambda} + \frac{e}{2\pi} A_\mu \partial_\nu a_\lambda \varepsilon^{\mu\nu\lambda}$$

Abelian Chern-Simons term

where $\nu = \frac{1}{m}$

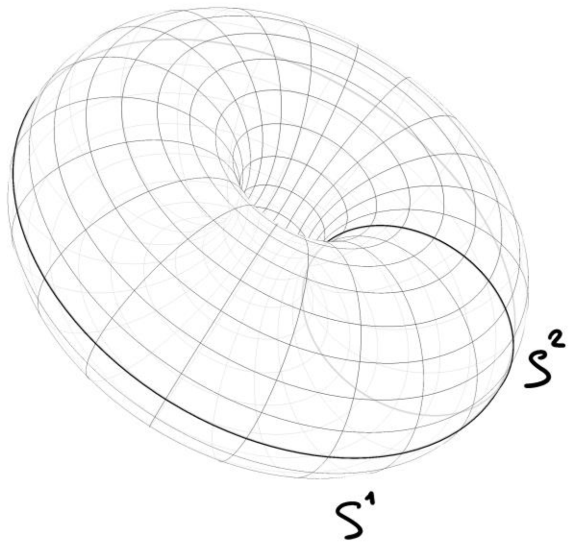
[X.-G. Wen, Quantum field theory of many-body systems: from the origin of sound to an origin of light and electrons (Oxford University Press on Demand, 2004).]



Topological Chern-Simons term

Chern-Simons term in (2+1)d:

$$\mathcal{L}_{\mathrm{CS}} = \frac{i\kappa}{2}\varepsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$



$$\kappa = \frac{n}{2\pi}$$ — Chern-Simons coupling

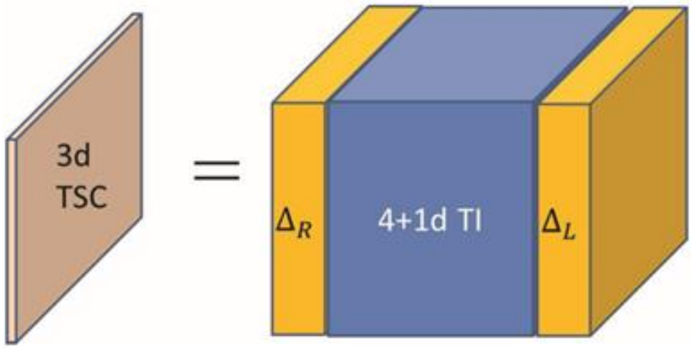
n —Chern-Simons level

Tasks we wanted so solve in the thesis

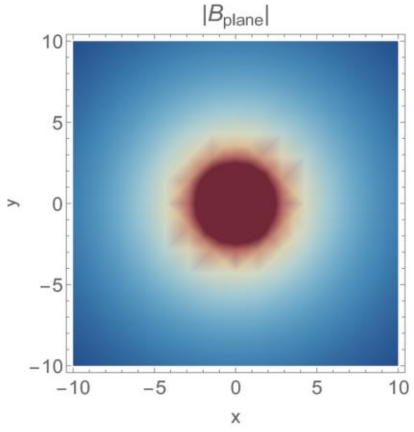
1. Solve classical field equations for a topological superconductor
2. Establish the existence of a deconfined quantum critical point in topological anisotropic ferromagnets
3. Obtain an exact bosonization duality for massless Dirac fermions

Topological London theory

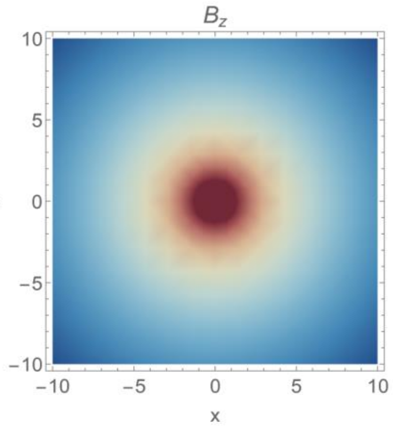
[Xiao-Liang Qi, Edward Witten, and Shou-Cheng Zhang, Phys. Rev. B **87**, 134519 (2013)]



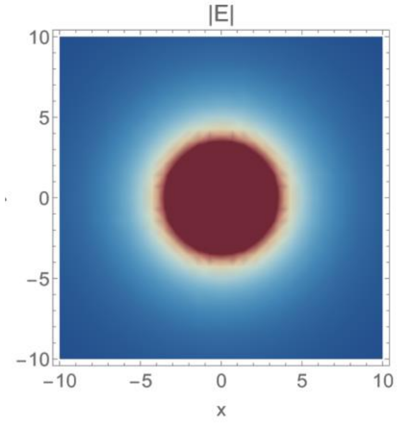
(3+1)d for one vortex



Plane components of the magnetic field



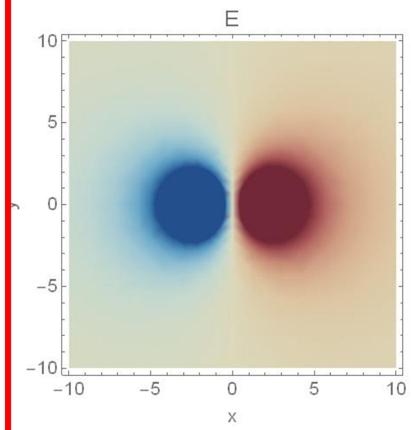
z-component of the magnetic field



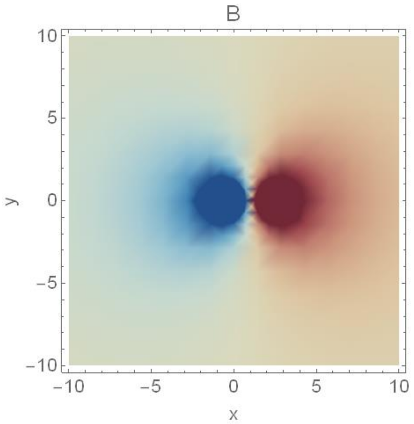
Amplitude of the electric field

- Static
- Long-wavelength approximation

(2+1)d for two vortices



Amplitude of the electric field



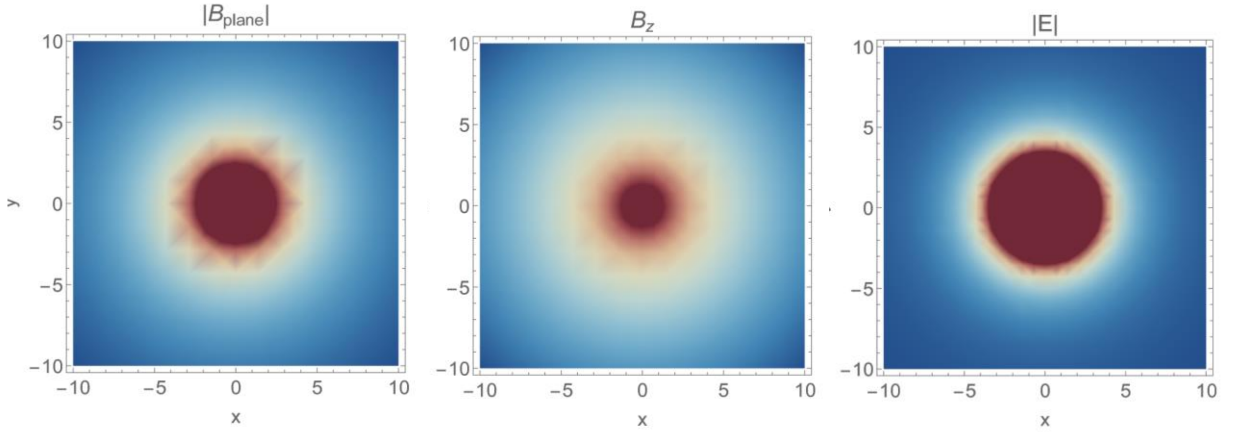
Magnetic field

Topological London theory

London screening — Witten effect

$$-\Delta\phi + \underbrace{M^2\phi}_{\text{London screening}} = \underbrace{-\frac{e^2}{8\pi^2}\nabla(\theta_L-\theta_R)\cdot\mathbf{B}}_{\text{Witten effect}}$$

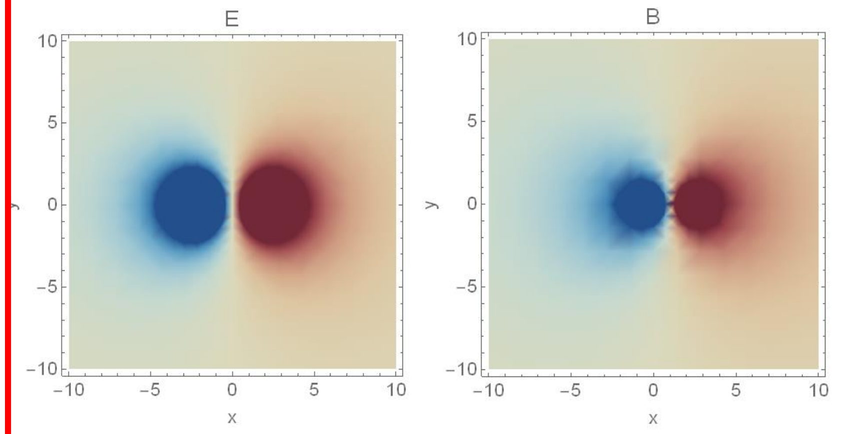
(3+1)d for one vortex



Plane components of the magnetic field — z-component of the magnetic field — Amplitude of the electric field

- Static
- Long-wavelength approximation

(2+1)d for two vortices



Amplitude of the electric field — Magnetic field

Topological London theory

London screening — Witten effect

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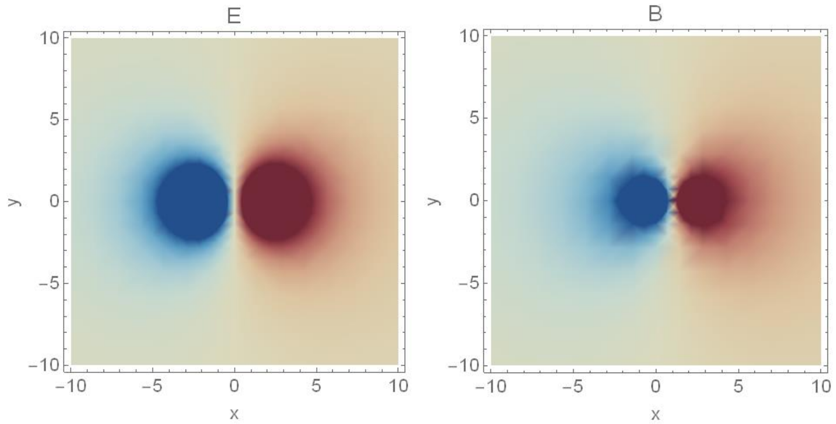
Anyonic statistics

(2+1)d for two vortices

Angular momentum

$$J = -\frac{\kappa m^2}{2\pi}$$

$$J = -\frac{\kappa\,(m+m_0)^2}{2\pi}$$



Amplitude of the electric field — Magnetic field

Deconfined quantum criticality and bosonization duality

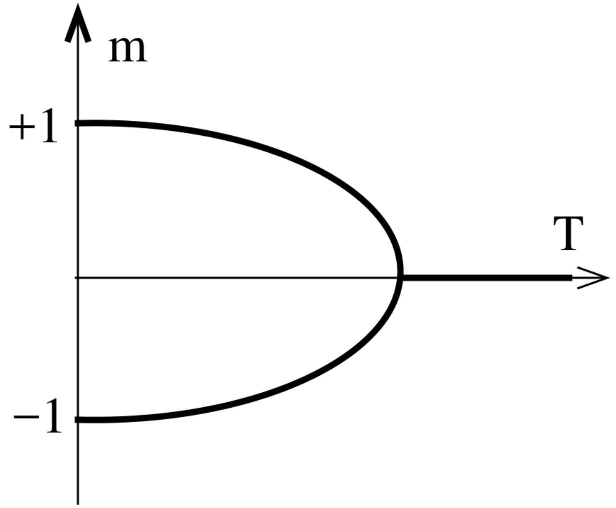
Landau-Ginzburg-Wilson paradigm of phase transitions

Effective free energy in Landau's theory

$$F(m) = F_0 + \frac{r}{2}m^2 + \frac{u}{4}m^4 + \ldots$$

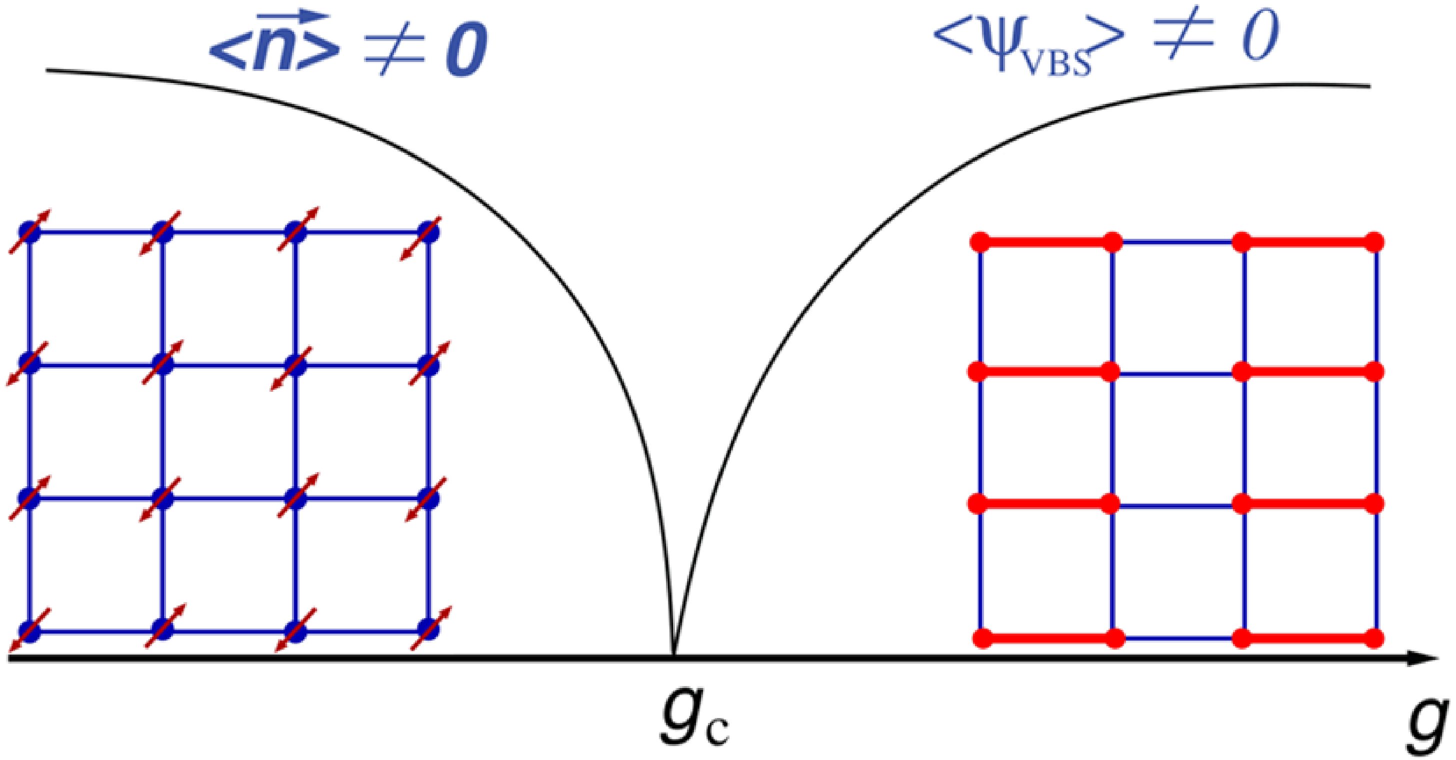
m — order parameter

Second order phase transition!



Deconfined quantum criticality

The magnetic Néel ground state



$|\bullet\!\!-\!\!\bullet\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

valence bond solid (VBS) quantum paramagnet

[T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science 303, 1490 (2004).]

Chern-Simons easy-plane antiferromagnets

CP[1] model — easy-plane anisotropy

$$S = \frac{1}{g}\int d^3x \left\{ \underbrace{\sum_{a=1,2} \left|(\partial_\mu - i a_\mu)\, z_a\right|^2}_{CP^1\ \text{model}} + \underbrace{\frac{K}{2g}\left(|z_1|^2 - |z_2|^2\right)^2}_{\text{easy-plane anisotropy}} \right\}$$

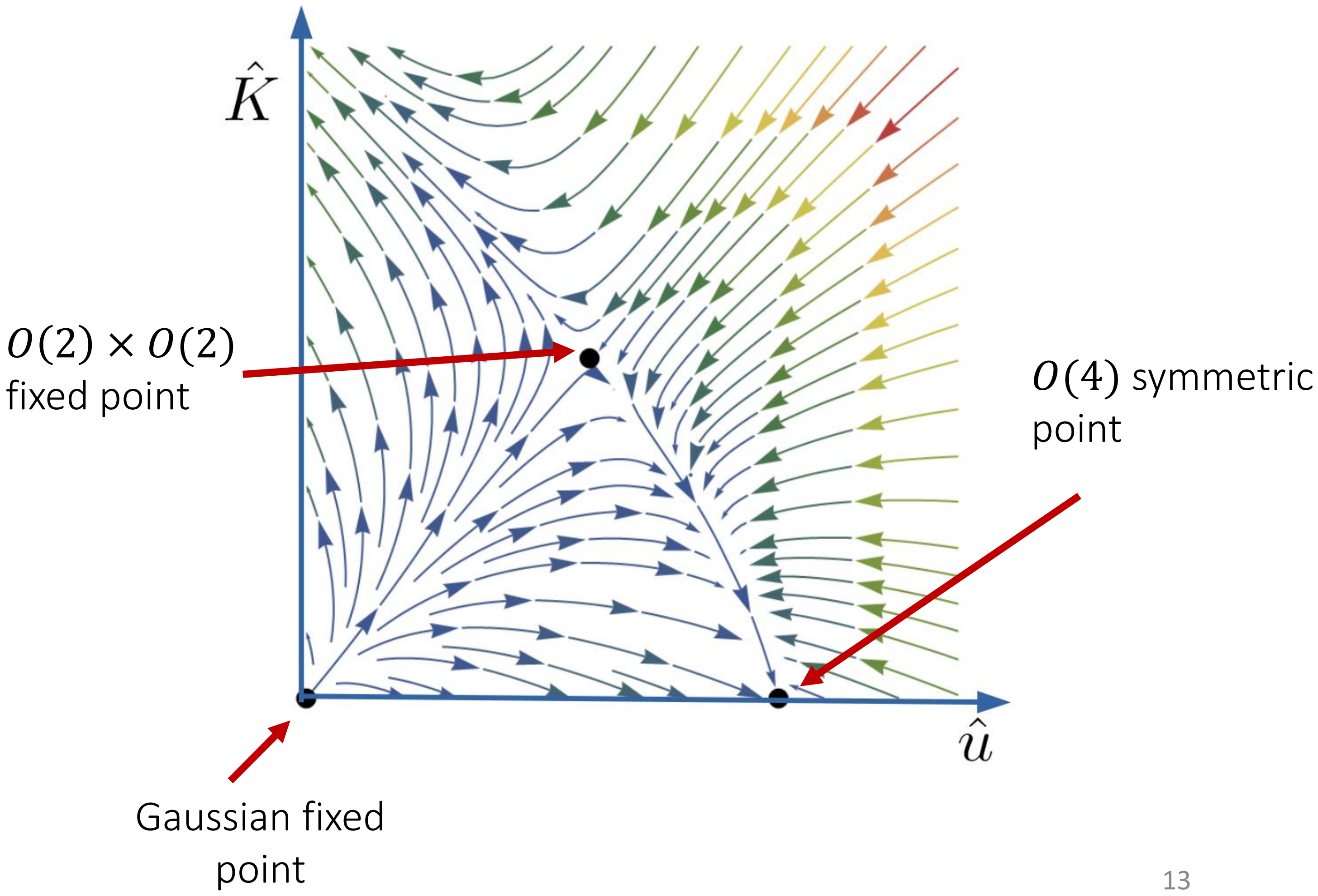
$$+ \int d^3x \left[\underbrace{\frac{i\kappa}{2}\varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda}_{CS\ \text{term}} + \underbrace{\frac{1}{2e^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda\right)^2}_{\text{Maxwell term}} \right]$$

CS term — Maxwell term

Renormalization group (RG) analysis: Existence of a critical point!

Chern-Simons easy-plane antiferromagnets feature a deconfined quantum critical point reached at $e^2 \to \infty$.

This is true for an arbitrary value of the Chern-Simons coupling.



Critical exponents — quantitative properties of the transition

$$\xi \sim \left|m_o^2 - m_c^2\right|^{-\nu}$$

Correlation length critical exponent

$$\langle \boldsymbol{n}(x) \cdot \boldsymbol{n}(0) \rangle \sim \frac{e^{\frac{-|x|}{\xi}}}{|x|^{1+\eta_N}}$$

Anomalous dimensions η

Quantization of critical exponents

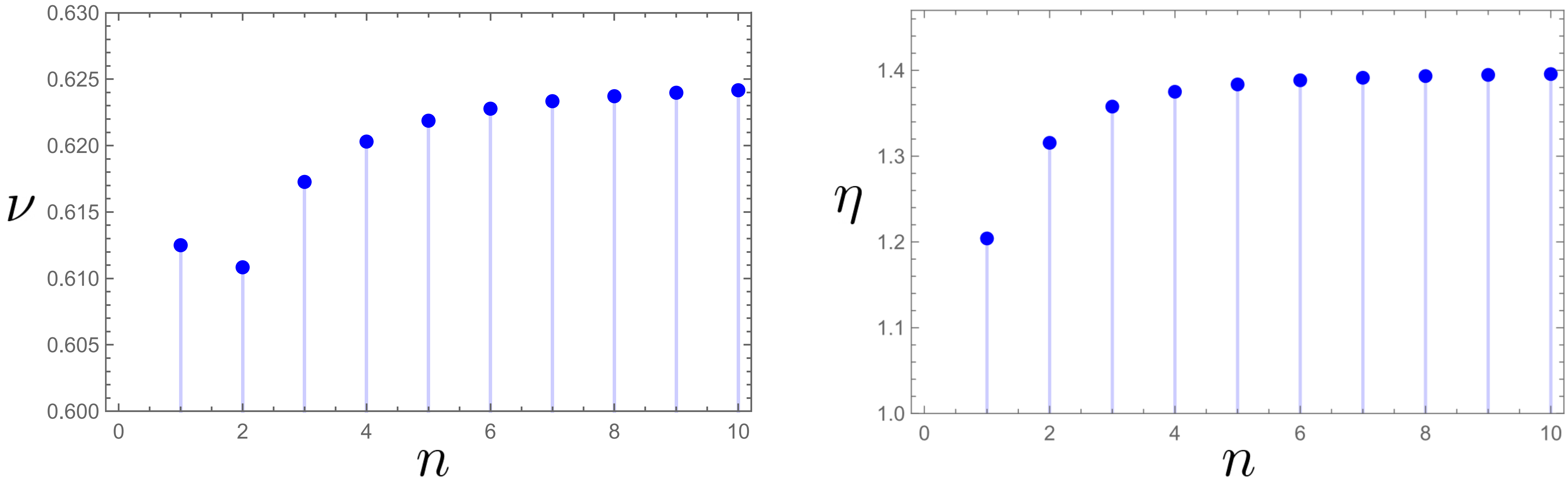
Calculated for level 1 CS term

$$\nu^{O(2)\times O(2)} = 49/80 \approx 0.613$$

$$\nu^{O(4)} = 2/3$$

$$\eta_N^{O(2)\times O(2)} = 59/49 \approx 1.2$$

$$\eta_N^{O(4)} = 164/147 \approx 1.12$$



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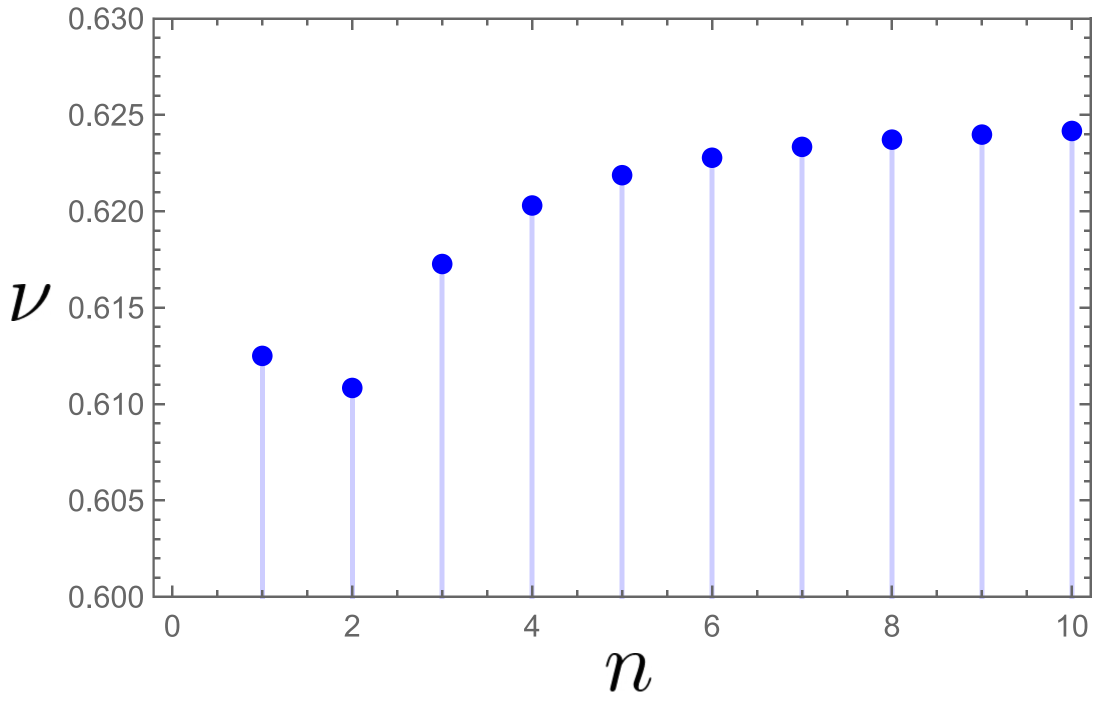
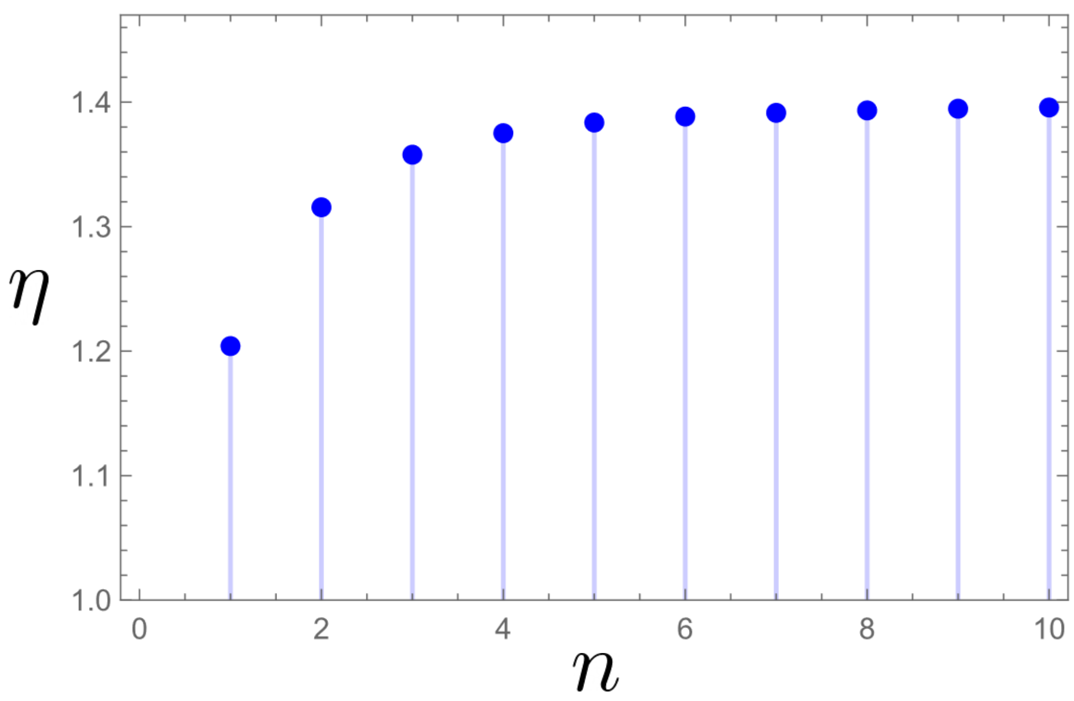
$$\eta_N^{O(4)} = 164/147 \approx 1.12$$

$$\eta^{O(2)} \approx 0.04$$

$$\eta^{O(3)} \approx 0.03$$





Field theory duality

Weakly coupled quantum field theory $\equiv$ Strongly coupled quantum field theory

Familiar examples : Bosonization, Jordan-Wigner, Particle-Vortex duality,…

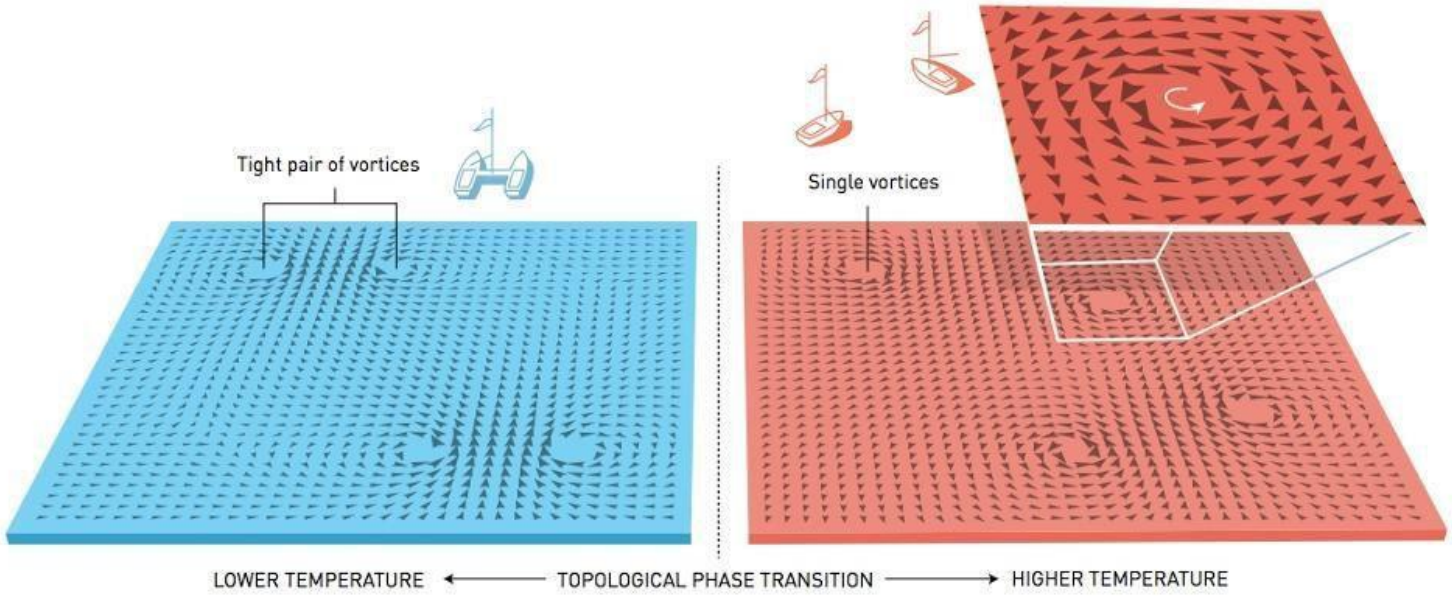
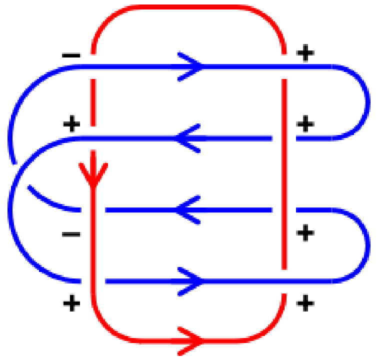


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

Berezinskii–Kosterlitz–Thouless (BKT) transition in 2d

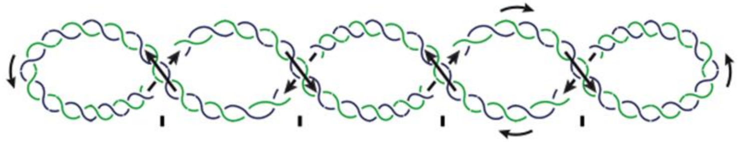
Dual theory at criticality

$$\widetilde{Z}_{\rm crit} = \sum_{\rm loops} \exp\left[i\frac{\pi\kappa}{2}\left(\sum_{a,b} 4\pi n_a n_b Lk_{ab} + \sum_a 4\pi n_a^2 Wr \right)\right]$$



Gauss linking number
Topological property — **Integer**!

Writhe
Geometrical property — **any** number



Bosonization duality for massless Dirac fermions

Dual theory at criticality at level 1 CS in the original model

$$\kappa = \frac{1}{2\pi}$$

$$\widetilde{Z}_{\text{crit}} = \sum_{\text{loops}} (-1)^{N_{ab}} e^{i\pi \sum_a n_a^2 \mathcal{W}_a}$$

$$N_{ab} = n_a n_b Lk_{ab}$$

Conclusions

1. We have obtained vortex solutions for classical field equations for a topological superconductor in the long-wavelength limit.
2. It was shown that topological easy-plane antiferromagnets undergo a second-order phase transition.
3. We predict quantized critical exponents for this distinct universality class.
4. We have established an explicit bosonization duality for massless Dirac fermions.

Thank you for your attention!