Large time and long distance asymptotics of the thermal correlators of the impenetrable anyonic lattice gas

Eduard Naichuk[1]

Supervisor
Nikolai Iorgov[2]

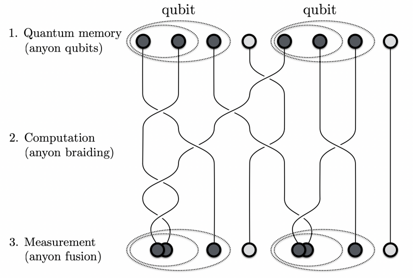
[1]Kyiv Academic University

[2]Bogolyubov Institute for Theoretical Physics

July 24, 2022

Motivation

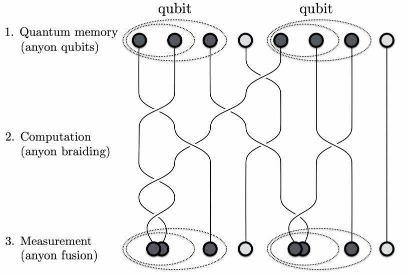
- Topological quantum computing



J. K. Pachos, Cambridge University Press, Cambridge, ISBN 9780511792908 (2012)

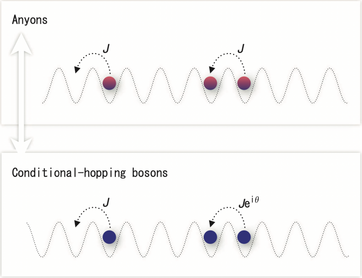
Motivation

- Topological quantum computing



J. K. Pachos, Cambridge University Press, Cambridge, ISBN 9780511792908 (2012)

- Systems of ultracold atoms



T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, Nature Communications 2, 361 (2011)

Bosons	Fermions
$[a_j, a_m^\dagger] = \delta_{jm}$	$\{a_j, a_m^\dagger\} = \delta_{jm}$
$[a_j, a_m] = 0$	$\{a_j, a_m\} = 0$
$[a_j^\dagger, a_m^\dagger] = 0$	$\{a_j^\dagger, a_m^\dagger\} = 0$

Bosons

$$[a_j, a_m^\dagger] = \delta_{jm}$$
$$[a_j, a_m] = 0$$
$$[a_j^\dagger, a_m^\dagger] = 0$$

Fermions

$$\{a_j, a_m^\dagger\} = \delta_{jm}$$
$$\{a_j, a_m\} = 0$$
$$\{a_j^\dagger, a_m^\dagger\} = 0$$

Anyons

$$a_j a_m^\dagger = \delta_{jm} - e^{-i\pi\kappa\epsilon(j-m)} a_m^\dagger a_j$$
$$a_j a_m = -e^{i\pi\kappa\epsilon(j-m)} a_m a_j$$
$$a_j^\dagger a_m^\dagger = -e^{i\pi\kappa\epsilon(j-m)} a_m^\dagger a_j^\dagger$$

Bosons

$$[a_j, a_m^\dagger] = \delta_{jm}$$
$$[a_j, a_m] = 0$$
$$[a_j^\dagger, a_m^\dagger] = 0$$

Fermions

$$\{a_j, a_m^\dagger\} = \delta_{jm}$$
$$\{a_j, a_m\} = 0$$
$$\{a_j^\dagger, a_m^\dagger\} = 0$$

Anyons

$$a_j a_m^\dagger = \delta_{jm} - e^{-i\pi\kappa\epsilon(j-m)} a_m^\dagger a_j$$
$$a_j a_m = -e^{i\pi\kappa\epsilon(j-m)} a_m a_j$$
$$a_j^\dagger a_m^\dagger = -e^{i\pi\kappa\epsilon(j-m)} a_m^\dagger a_j^\dagger$$

$\epsilon(m) = m/|m|, \quad \epsilon(0) = 0,$

$\kappa \in [0, 1]$ – statistics parameter.

Anyonic model

$$H = -\sum_{j=1}^{L} \frac{1}{2}(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) + h \sum_{j=1}^{L} a_j^\dagger a_j,$$

$$a_{L+1} = a_L, \qquad a_{L+1}^\dagger = a_L^\dagger.$$

L – number of lattice sites

h – chemical potential

Anyonic model

Two-point correlation function ($L \to \infty$):

$$G(x,t) = \frac{\mathrm{Tr}[e^{-\beta H} a^\dagger_{x+1}(t) a_1(0)]}{\mathrm{Tr}[e^{-\beta H}]},$$

where $\beta = 1/T, \quad a^\dagger_x(t) = e^{iHt} a^\dagger_x e^{-iHt}.$

Anyonic model

Two-point correlation function ($L \to \infty$):

$$G(x,t) = \frac{\mathrm{Tr}[e^{-\beta H} a^{\dagger}_{x+1}(t) a_1(0)]}{\mathrm{Tr}[e^{-\beta H}]} = \begin{array}{c}\text{Fredholm}\\ \text{determinants}\end{array} \begin{pmatrix} & x & t & \\ \kappa & & \beta & & h \end{pmatrix}$$

Anyonic model

Two-point correlation function ($L \to \infty$):

$$G(x,t) = \frac{\mathrm{Tr}[e^{-\beta H} a^{\dagger}_{x+1}(t) a_1(0)]}{\mathrm{Tr}[e^{-\beta H}]} = \begin{array}{c}\text{Fredholm}\\ \text{determinants}\end{array} \begin{pmatrix} & x & & t & \\ \kappa & & \beta & & h \end{pmatrix}$$

- For large x and t it is difficult to compute Fredholm determinants numerically;

Anyonic model

Two-point correlation function ($L \to \infty$):

$$G(x,t) = \frac{\mathrm{Tr}[e^{-\beta H} a^{\dagger}_{x+1}(t) a_1(0)]}{\mathrm{Tr}[e^{-\beta H}]} = \begin{matrix}\text{Fredholm}\\ \text{determinants}\end{matrix} \begin{pmatrix} & x & t & \\ \kappa & \beta & h \end{pmatrix}$$

- For large x and t it is difficult to compute Fredholm determinants numerically;
- One need to find more effective ways to study asymptotics.

Effective form factor approach

To specify the effective form factor we require two smooth periodic functions $\nu(k)$, $g(k)$. Here L is regarded as a system size.

- The first one is called the effective phase shift and defines the *shifted* set of momenta as solutions of

$$e^{ikL} = e^{-2\pi i \nu(k)}, \quad e^{iqL} = 1.$$

Effective form factor approach

To specify the effective form factor we require two smooth periodic functions $\nu(k)$, $g(k)$. Here L is regarded as a system size.

- The first one is called the effective phase shift and defines the *shifted* set of momenta as solutions of

$$e^{ikL} = e^{-2\pi i \nu(k)}, \quad e^{iqL} = 1.$$

- The second function is in the definition of effective form factors

$$|\langle \mathbf{k}|\mathrm{q}^{(a)}\rangle|^2 = L^{1-2L} \prod_{j=1}^{L} \frac{e^{g(k_j)-g(q_j)} \sin^2 \pi\nu(k_j)}{1+\frac{2\pi}{L}\nu'(k_j)} e^{g(q_a)} \det{}^2 D^a,$$

where

$$\det D^a = \begin{vmatrix} \cot\frac{k_1-q_1}{2} & \dots & \cot\frac{k_L-q_1}{2} \\ \vdots & \ddots & \vdots \\ \cot\frac{k_1-q_L}{2} & \dots & \cot\frac{k_L-q_L}{2} \\ 1 & \dots & 1 \end{vmatrix},$$

$$\mathrm{q}^{(a)} = \{q_1, \dots, q_{a-1}, q_{a+1}, \dots, q_L\}, \quad {\scriptstyle a=1,\dots,L}.$$

Effective form factor approach

The tau (correlation) function is defined as series over these form factors

$$\tau(x,t) = \sum_{\mathrm{q}^a} |\langle \mathrm{k}|\mathrm{q}^a\rangle|^2 e^{-ix(P(\mathrm{k})-P(\mathrm{q}^a))+it(E(\mathrm{k})-E(\mathrm{q}^a))},$$

where

$$P(\mathrm{q}) = \sum_{q\in\mathrm{q}} q, \quad E(\mathrm{q}) = \sum_{q\in\mathrm{q}} \varepsilon(q), \quad \varepsilon(q) = h - \cos q.$$

Effective form factor approach

$$\tau(x,t) = \begin{array}{c}\text{Fredholm}\\ \text{determinants}\end{array} \begin{pmatrix} & x \quad t & \\ g(q) & \nu(q) & h \end{pmatrix}$$

Effective model
g(q) ν(q) h

Anyonic model
κ β h

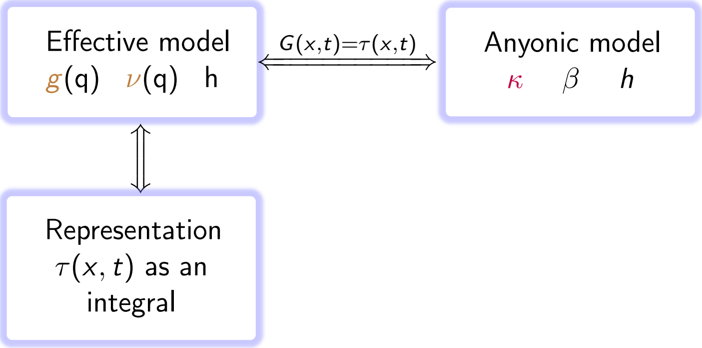
Effective form factor approach

$$\tau(x,t) = \text{Fredholm determinants}\begin{pmatrix} & x \quad t & \\ g(q) & \nu(q) & h \end{pmatrix}$$

Effective model	$\overset{G(x,t)=\tau(x,t)}{\Longleftrightarrow}$	Anyonic model
$g(q) \quad \nu(q) \quad h$		$\kappa \quad \beta \quad h$

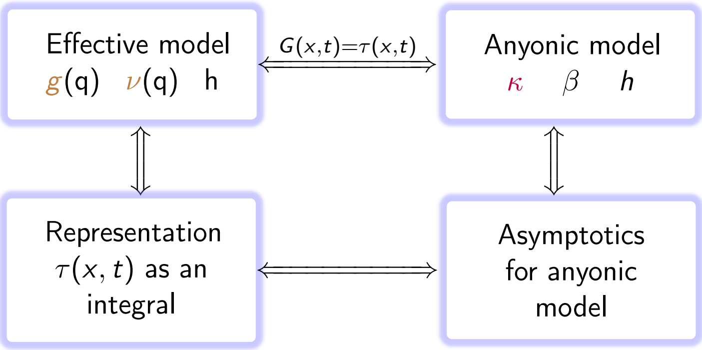
Effective form factor approach

$$\tau(x,t) = \begin{array}{c}\text{Fredholm}\\ \text{determinants}\end{array} \left({}_{g(q)} \quad {}^{x} \quad \nu(q) \quad {}^{t} \quad h \right)$$



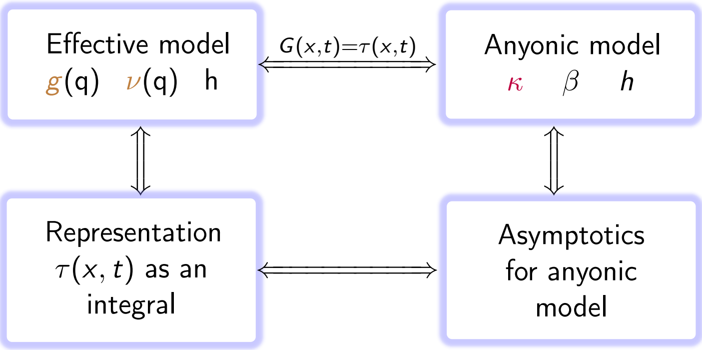
Effective form factor approach

$$\tau(x,t) = \text{Fredholm determinants}\begin{pmatrix} & x \quad t & \\ g(q) & \nu(q) & h \end{pmatrix}$$



Effective form factor approach

$$\tau(x,t) = \begin{matrix}\text{Fredholm}\\ \text{determinants}\end{matrix} \begin{pmatrix} & x \quad t & \\ g(q) & \nu(q) & h \end{pmatrix}$$



O. Gamayun, N. Iorgov, and Y. Zhuravlev, Effective free-fermionic form factors and the XY spin chain, SciPostPhys. **10**, 70 (2021).

Results

Space-like region $(x > t)$, $x \gg 1$, $t \gg 1$

$$G(x,t) \approx C_2 K(x,t) e^{-x \log z_0 + \frac{t\pi}{\beta}(1-\kappa)}$$

where $K(x,t)$ and z_0 are given by

$$K(x,t) = Z^2[\nu] e^{ix \int_{-\pi}^{\pi} \nu(q) dq}, \quad \nu(q) = \nu_+(q),$$

$$\nu_\pm(q) = \pm \frac{1}{2\pi i} \log\left(1 + n_F(q)(e^{\pm i\pi\kappa} - 1)\right),$$

$$n_F(q) = \frac{1}{e^{\beta\varepsilon(q)} + 1}, \quad \varepsilon(q) = h - \cos q,$$

$$z_0 = h_0 + \sqrt{h_0^2 - 1}, \quad h_0 = h + \frac{i\pi}{\beta}(1 - \kappa).$$

The prefactors $Z^2[\nu]$ and C_2 are constants for fixed x/t.

Results

Time-like region ($x < t$), $x \gg 1$, $t \gg 1$

$$G(x,t) \approx R_\infty t^{-\delta_1^2-\delta_2^2} e^{i\int_{-\pi}^{\pi}(x-t\varepsilon'(q))\nu(q)dq} \times \left(\frac{a_1 e^{-ixq_1+it\varepsilon(q_1)}}{t^{\frac{1}{2}+\delta_1}} + \frac{a_2 e^{-ixq_2+it\varepsilon(q_2)}}{t^{\frac{1}{2}+\delta_2}} \right)$$

The critical momenta q_1 and q_2 are defined by

$$q_1 = \arcsin(x/t), \qquad q_2 = \pi - \arcsin(x/t),$$

the effective phase shift $\nu(q)$ is piecewise function

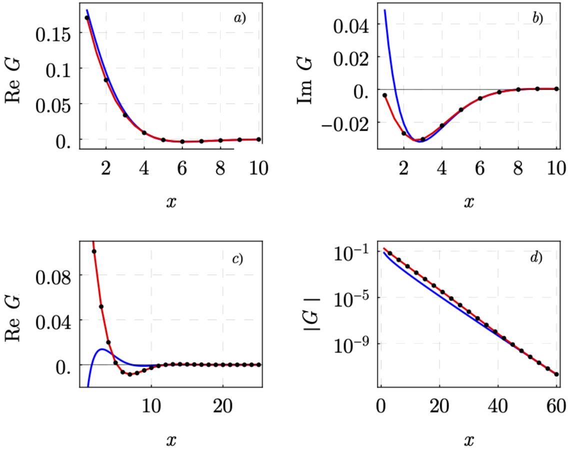
$$\nu(q) = \begin{cases} \nu_+(q) & \text{if } -\pi < q < q_1 \text{ or } q_2 < q \leq \pi, \\ \nu_-(q) & \text{if } q_1 < q < q_2, \end{cases}$$

and δ_1 and δ_2 are the magnitudes of jumps of $\nu(q)$ at critical momenta

$$\delta_1 = \nu_-(q_1) - \nu_+(q_1), \quad \delta_2 = \nu_+(q_2) - \nu_-(q_2).$$

Results

Asymptotics for space-like region



Black dots – numerical values of Fredholm determinants, blue lines – asymptotics.
Panels *a*) and *b*) correspond to $x/t = 2.5$, panels *c*) and *d*) correspond to $x/t = 1.3$.
$\kappa = 0.6$, $h = 0.7$, $\beta = 2.3$.

Y. Zhuravlev, E. Naichuk, N. Iorgov and O. Gamayun, Phys. Rev. B **105**, 085145 (2022)

Conclusions

- Asymptotic behaviour of the correlation function at large time and long distance in both space-like and time-like regions was derive;
- It was found that on top of the exponential decay the additional power factor appears in the time-like region.

Conclusions

- Asymptotic behaviour of the correlation function at large time and long distance in both space-like and time-like regions was derive;
- It was found that on top of the exponential decay the additional power factor appears in the time-like region.

Y. Zhuravlev, E. Naichuk, N. Iorgov and O. Gamayun, Large-time and long-distance asymptotics of the thermal correlators of the impenetrable anyonic lattice gas, Phys. Rev. B **105**, 085145 (2022)

Thank you for your attention!